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Boundary-layer flow of a micropolar fluid on a continuously moving or fixed permeable surface

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Abstract

The present paper deals with the analysis of the boundary-layer flow of a micropolar fluid on a fixed or continuously moving permeable surface. Both parallel and reverse moving surfaces to the free stream are considered. The resulting system of non-linear ordinary differential equations is solved numerically using the Keller-box method. Numerical results are obtained for the skin friction coefficient and the local Nusselt number for some values of the parameters, namely the velocity ratio parameter, suction/injection parameter and material parameter, while the Prandtl number is fixed to be unity. The results indicate that dual solutions exist when the plate and the free stream move in opposite directions.

Keywords: Boundary-layer; Dual solutions; Micropolar fluid; Moving surface; Suction/injection

1. Introduction

The theory of microfluids, as developed by Eringen [1] has been a field of active research for the last few decades as this class of fluids represents mathematically many industrially important fluids, like paints, blood, body fluids, polymers, colloidal fluids and suspension fluids. This theory takes into account the initial characteristics of the substructure particles which are allowed to undergo rotation. Micropolar fluids are fluids with microstructure belonging to a class of fluids with non-symmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium. Takhar and Soundalgekar [2] studied the effects of suction and injection on the flow past a continuously moving semi-infinite porous plate in a micropolar fluid at rest. In a recent paper, Ishak et al. [3] analyzed the problem of fluid flow on a continuously moving plate immersed in a moving micropolar fluid, by considering an impermeable plate. The objective of this study is therefore, to extend the work of Ishak et al. [3] by considering the effects of suction and injection on the flow and heat transfer characteristics of a moving plate in a moving fluid. Both parallel and reverse moving plates to the free stream are considered. In this respect, we follow Afzal et al. [4] by employing a composite velocity instead of considering two cases separately, where the velocity of the moving plate is greater or less than the free stream velocity.

2. Problem formulation and basic equations

Consider the steady two-dimensional laminar flow of an incompressible micropolar fluid due to a moving plate with a constant velocity U_w in the same or opposite direction to the mainstream of constant velocity U_∞. The plate emerges from the slot of an extrusion die, as shown in Fig. 1. The origin of the Cartesian coordinate system is placed at the location where the plate is drawn into the fluid medium with the x-axis measured along the plate in the right direction and the

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Nomenclature

A	dimensionless constant of integration
C_f	skin friction coefficient
f	dimensionless stream function
f_0	suction/injection parameter
g	dimensionless microinertia
h	dimensionless angular velocity
j	microinertia density
k	thermal conductivity
K	material parameter
n	constant
N	angular velocity
Nu_x	local Nusselt number
Pr	Prandtl number
q_w	heat transfer from the plate
Re_w	local Reynolds number, $U_w x/\nu$
Re_∞	local Reynolds number, $U_\infty x/\nu$
T	fluid temperature
T_w	plate temperature
T_∞	ambient temperature
u, v	velocity components along the x- and y-directions, respectively
U_w	plate velocity
U_∞	ambient fluid velocity
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
γ	spin-gradient viscosity
κ	vortex viscocity
η	similarity variable
λ	velocity ratio parameter, $\lambda = U_w/(U_w + U_\infty)$
θ	dimensionless temperature
ν	kinematic viscosity
μ	dynamic viscosity
ρ	fluid density
τ_w	skin friction
ψ	stream function

Subscripts

w	condition at the wall
∞	ambient condition

Superscript

$'$	differentiation with respect to η

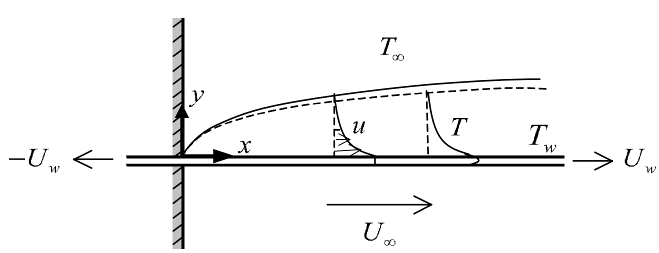

Fig. 1. Physical model and coordinate system.

y-axis is measured normal to the plate. Neglecting external body forces and the viscous dissipation effects, the system of equations governing the problem under consideration, within the boundary-layer approximations is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(\frac{\mu + \kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y}, \tag{2}$$

$$\rho j\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \frac{\partial}{\partial y}\left(\gamma\frac{\partial N}{\partial y}\right) - \kappa\left(2N + \frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial j}{\partial x} + v\frac{\partial j}{\partial y} = 0, \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

subject to the boundary conditions:

$$\begin{aligned} &u = U_w, \quad v = V_w, \quad j = J_w, \quad N = -n\frac{\partial u}{\partial y}, \quad T = T_w \quad \text{at } y = 0, \\ &u \to U_\infty, \quad N \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty. \end{aligned} \tag{6}$$

Here u and v are the velocity components along the x- and y-axes, respectively, N is the angular velocity or microrotation whose direction of rotation is in the x–y plane, T, T_w, T_∞, ρ, μ, κ, γ, ν, j and α are the fluid temperature, plate temperature, ambient fluid temperature, fluid density, dynamic viscosity, vortex viscosity, spin-gradient viscosity, kinematic viscosity, microinertia density and thermal diffusivity, respectively. Further, $V_w(x) < 0$ and $V_w(x) > 0$ are for mass suction and mass injection, respectively. We notice that n is a constant such that $0 \leqslant n \leqslant 1$, where the case $n = 0$ is called strong concentration by Guram and Smith [5], indicates $N = 0$ near the wall and represents concentrated particle flows in which the microelements close to the wall surface are unable to rotate (Jena and Mathur [6]). The case $n = 1/2$ indicates the vanishing of anti-symmetrical part of the stress tensor and denotes weak concentration (Ahmadi [7]). The case $n = 1$, as suggested by Peddieson [8], is used for the modeling of turbulent boundary-layer flows. However, we shall consider here only the case of weak concentration of particles at the plate

($n = 1/2$). In order that similarity solutions of Eqs. (1)–(5) subject to the boundary conditions (6) exist, we take:

$$J_w(x) = ax, \quad V_w(x) = -\left[\frac{\nu(U_w + U_\infty)}{2x}\right]^{1/2} f_0, \tag{7}$$

where a and f_0 are constants. We notice that f_0 determines the transpiration rate at the surface, with $f_0 > 0$ for suction, $f_0 < 0$ for blowing or injection, and $f_0 = 0$ corresponds to an impermeable plate.

We follow the work of many recent authors by assuming that γ is given by (cf. Ahmadi [7] or Kline [9]):

$$\gamma = (\mu + \kappa/2)j = \mu(1 + K/2)j, \tag{8}$$

where $K = \kappa/\mu$ denotes the dimensionless viscosity ratio and K is called the material parameter. This assumption is invoked to allow the field of equations predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity. Eq. (8) also has been used by Gorla [10] and Ishak et al. [11] to study different problems of convective flow of micropolar fluids. It is stated by Ahmadi [7] that for non-constant microinertia it is possible using Eq. (8) to find similar and self-similar solutions for a large number of problems of micropolar fluids. It is also worth mentioning that the case $K = 0$ describes the classical Navier–Stokes equations for a viscous and incompressible fluid.

We introduce now the following similarity variables:

$$\begin{gathered}
\psi = (2\nu x)^{1/2}(U_w + U_\infty)^{1/2} f(\eta), \\
N = \left(\frac{1}{2\nu x}\right)^{1/2}(U_w + U_\infty)^{3/2} h(\eta), \\
\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad j = \left(\frac{2\nu x}{U_w + U_\infty}\right) g(\eta), \\
\gamma = \left(\mu + \frac{\kappa}{2}\right)\left(\frac{2\nu x}{U_w + U_\infty}\right) g(\eta), \quad \eta = \left(\frac{U_w + U_\infty}{2\nu x}\right)^{1/2} y,
\end{gathered} \tag{9}$$

where ψ is the stream function defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ so as to identically satisfy Eq. (1). Substituting variables (9) into Eqs. (2)–(5), we obtain the following ordinary differential equations:

$$(1 + K)f''' + ff'' + Kh' = 0, \tag{10}$$

$$(1 + K/2)(gh')' + g(fh)' - K(2h + f'') = 0, \tag{11}$$

$$2gf' - fg' = 0, \tag{12}$$

$$\theta'' + Pr f\theta' = 0, \tag{13}$$

and the boundary conditions (6) becomes:

$$\begin{gathered}
f(0) = f_0, \quad g(0) = g_0, \quad f'(0) = \lambda, \\
h(0) = -nf''(0), \quad \theta(0) = 1, \\
f'(\infty) \to 1 - \lambda, \quad h(\infty) \to 0, \quad \theta(\infty) \to 0,
\end{gathered} \tag{14}$$

where g_0 is a constant, Pr is the Prandtl number and λ is the velocity ratio parameter defined as:

$$\lambda = \frac{U_w}{U_w + U_\infty}, \tag{15}$$

with $U_w + U_\infty \neq 0$. We notice that $\lambda = 0$ and 1 correspond to a fixed plate in a moving fluid and a moving plate in a quiescent fluid, respectively. The case $0 < \lambda < 1$ is when the plate and the fluid move in the same direction. If $\lambda < 0$, the free stream is directed towards the positive x-direction, while the plate moves towards the negative x-direction. If $\lambda > 1$, the free stream is directed towards the negative x-direction, while the plate moves towards the positive x-direction. However, in this paper we consider only the case of $\lambda \leqslant 1$, i.e. the direction of the free stream is fixed (towards the positive x-direction).

The solution of Eq. (12) satisfying the boundary conditions (14) is given by:

$$g = Af^2, \tag{16}$$

where A is a dimensionless constant of integration. If $K \neq 0$, but $A = 0$, from Eq. (11), we get:

$$h = -\frac{1}{2}f'', \tag{17}$$

that is the gyration is identical to the angular velocity. Using relation (17), Eq. (10) becomes:

$$(1 + K/2)f''' + ff'' = 0, \tag{18}$$

which can be reduced to the Blasius equation with a simple change of the dependent variables.

The physical quantities of interest are the skin friction coefficient and the local Nusselt number, which are defined as:

$$C_f = \frac{\tau_w}{\rho(U_w + U_\infty)^2/2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \tag{19}$$

where the skin friction τ_w and the heat transfer from the plate q_w are given by:

$$\tau_w = \left[(\mu + \kappa)\frac{\partial u}{\partial y} + \kappa N\right]_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{20}$$

Using the similarity variables (9), we get:

$$\begin{gathered}
\frac{1}{2}C_f(Re_w + Re_\infty)^{1/2} = \frac{1}{\sqrt{2}}(1 + K/2)f''(0), \\
Nu_x/(Re_w + Re_\infty)^{1/2} = -\frac{1}{\sqrt{2}}\theta'(0),
\end{gathered} \tag{21}$$

where $Re_w = U_w x/\nu$ and $Re_\infty = U_\infty x/\nu$ are the local Reynolds numbers.

3. Results and discussion

The non-linear ordinary differential equations (10), (11), (13) and (16), satisfying the boundary conditions (14) are solved numerically using the Keller-box method for several values of the parameters, namely the material parameter K, velocity ratio parameter λ and suction/injection parameter f_0, while the Prandtl number $Pr = 1$, the constant $A = 1$

Table 1
Values of $f''(0)/\sqrt{2}$ when $K = 0$ and $f_0 = 0$

λ	Blasius [12]	Howarth [13]	Sakiadis [14]	Cortell [15]	Present results
0	0.332	0.33206		0.33206	0.3321
0.5					0
1			−0.44375		−0.4438

and $n = 1/2$ (weak concentration of fluid particles at the plate). The numerical results compared well with those obtained by previous investigations, as shown in Table 1.

The variations of the skin friction coefficient $f''(0)$ and the local Nusselt number $-\theta'(0)$ as a function of λ for various values of the material parameter K are shown in Figs. 2 and 3, respectively. It is evident that, in each figure, all curves intersect at a point where $\lambda = 0.5$; that is when the plate and the fluid move with the same velocity. In this case, $f''(0) = 0$ since the skin friction $\tau_w = 0$, but $-\theta'(0) = 0.5641 \neq 0$ which implies that the heat transfer still occurs from the plate to the fluid even when they are moving with the same velocity. This is because the plate and the fluid are at different temperature. The zero skin friction in this case does not mean separation. The values of $f''(0)$ are positives when $\lambda < 0.5$, while they are negatives when $\lambda > 0.5$. Physically, positive sign of $f''(0)$ implies that the fluid exerts a drag force on the plate and negative sign implies the opposite. Moreover, the absolute value of $f''(0)$ decreases when K increases. Thus, micropolar fluids show drag reduction compared to Newtonian fluids.

Figs. 4 and 5 present the effect of suction/injection parameter f_0 on the skin friction coefficient $f''(0)$ when $K = 1$, and the local Nusselt number $-\theta'(0)$ when $Pr = 1$ and $K = 1$. We notice that the Prandtl number Pr gives no effect to the skin friction coefficient, as can be seen from Eqs. (10)–(13). As expected, the absolute value of $f''(0)$ is larger for suction compared to injection. The similar trend is observed for the effect of f_0 on the local Nusselt number. Thus, injection can be introduced to reduce the drag force and in consequence reduces the heat transfer rate at the surface. Fig. 4 also shows that the skin friction is zero when

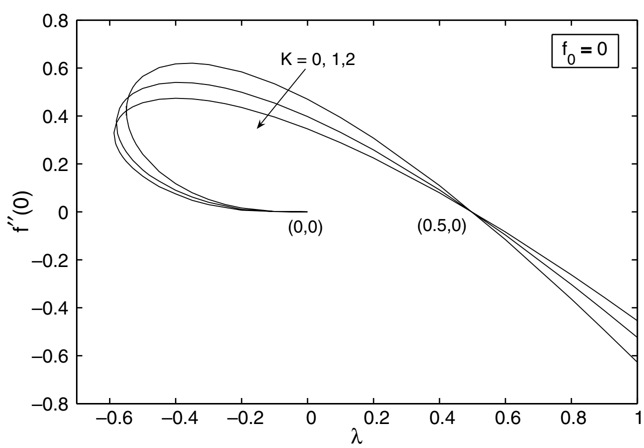

Fig. 2. Skin friction coefficient $f''(0)$ as a function of λ for various values of K when $f_0 = 0$.

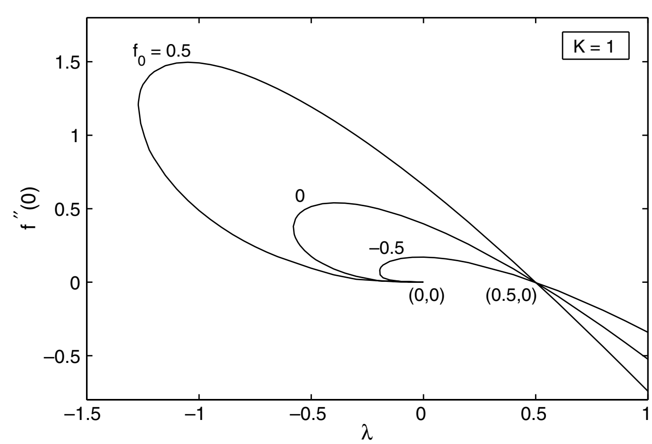

Fig. 4. Skin friction coefficient $f''(0)$ as a function of λ for various values of f_0 when $K = 1$.

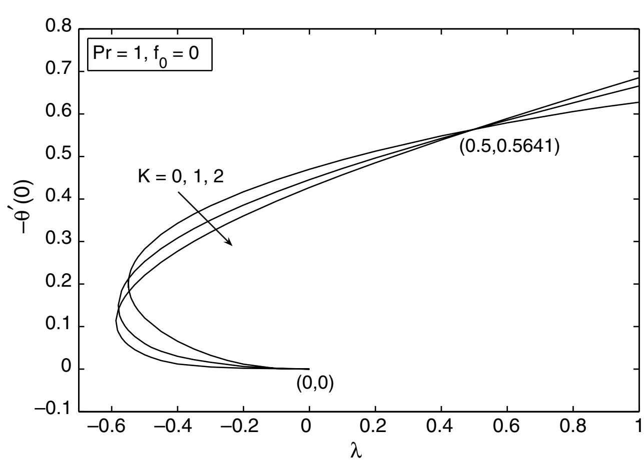

Fig. 3. Local Nusselt number $-\theta'(0)$ as a function of λ for various values of K when $f_0 = 0$.

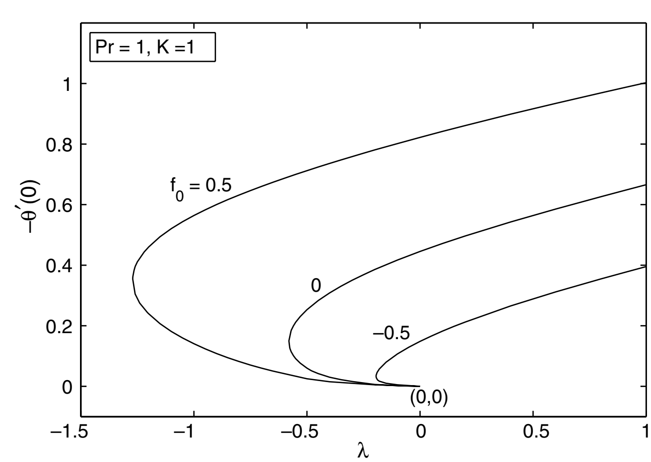

Fig. 5. Local Nusselt number $-\theta'(0)$ as a function of λ for various values of f_0 when $K = 1$.

the plate and the fluid move with the same velocity, a consistent result with those stated above.

Figs. 2–5 show the existence of dual solutions when $\lambda < 0$. The solution for a particular value of K or f_0 exists up to a critical value of λ (λ_c say). Beyond this value, the boundary-layer separates from the surface, thus we are unable to get the solution using the boundary-layer approximations. To obtain further solution, the full Navier–Stokes equations have to be used. It is evident from these figures that larger values of K or f_0 delay the boundary-layer separation. Thus, the boundary-layer separation is delayed for micropolar fluid or by introducing suction. The curve bifurcates at $\lambda = \lambda_c$, and the lower branch solution continues further and terminates at $(0, 0)$ for both $f''(0)$ and $-\theta'(0)$. The samples of velocity, microrotation (angular velocity) and temperature profiles which show the existence of dual solutions, are given in Figs. 6–8, respectively. As can be seen from Fig. 7, the lower branch solution is unstable and does not correspond to a physically realizable situation. However, the results (lower branches) are of interest in so far as the differential equations are concerned, since they may appear in other situations where the corresponding solutions could have more realistic meaning (see

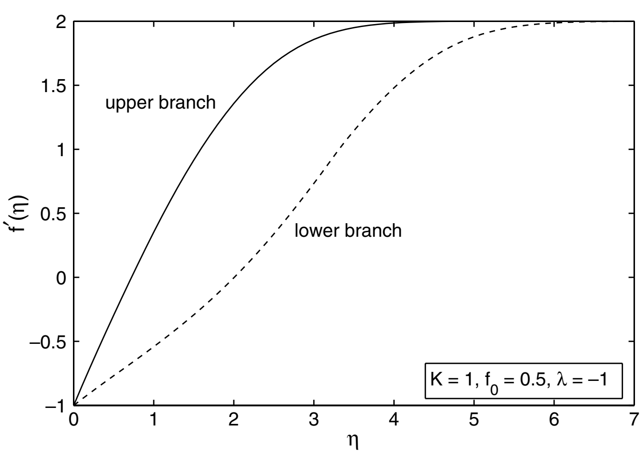

Fig. 6. Velocity profiles for $K = 1$, $f_0 = 0.5$ and $\lambda = -1$.

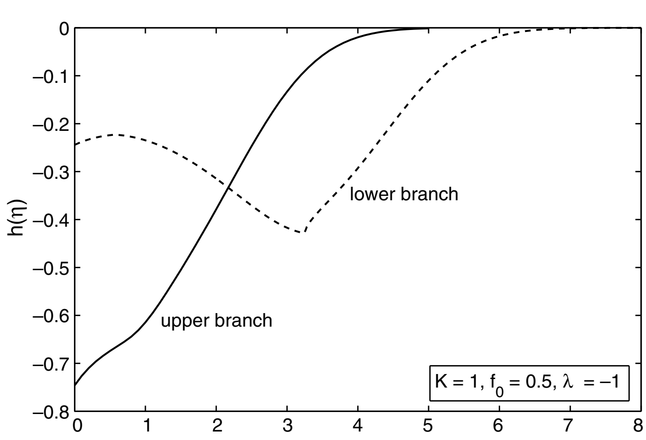

Fig. 7. Angular velocity profiles for $K = 1$, $f_0 = 0.5$ and $\lambda = -1$.

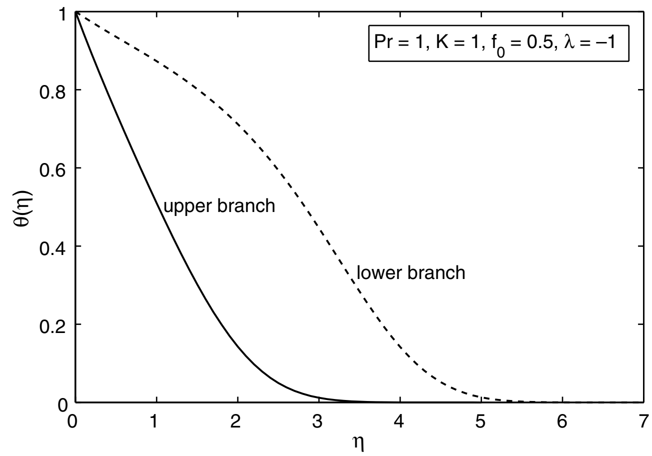

Fig. 8. Temperature profiles for $K = 1$, $f_0 = 0.5$ and $\lambda = -1$.

Merkin and Ingham [16] and Ridha [17]). The existence of dual solutions in the neighborhood of the separation region gives an early sign that the flow is unstable and in transition to become turbulent.

4. Conclusions

In this paper, we have theoretically studied the problem of steady boundary-layer flow of a micropolar fluid on a continuously moving or fixed permeable surface. The governing boundary-layer equations have been solved numerically using the Keller-box method. The numerical results for the skin friction coefficient and the local Nusselt number have been obtained and illustrated in graphs. A discussion of the effect of material parameter K, velocity ratio parameter λ and suction/injection parameter f_0, while the Prandtl number $Pr = 1$, on the skin friction and heat transfer rate at the surface in the case of weak concentration particles at the plate ($n = 1/2$) has been done. It has been demonstrated that dual solutions exist when the plate and the free stream move in the opposite directions. Moreover, micropolar fluids show drag reduction characteristic compared to classical Newtonian fluids, and the boundary-layer separation is delayed for micropolar fluids or by introducing suction.

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